



AN REAL TIME APPLICATION OF FUZZY WEIBULL DISTRIBUTION FOR THE CROP PRODUCTION IN AGRICULTURAL PROBLEM

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Abstract:

This study proposes an investigation into dependability characteristics in fuzzy situations. It is used to describe the fuzzy Weibull distribution and the lifetimes of components. We offer the fuzzy reliability and hazard function formulas, as well as their α -cut set. Finally, numerical examples are provided to demonstrate how to derive fuzzy reliability characteristics and the α -cut set. The real-time agriculture data set from Karnataka in 2018 is analyzed to generate plots for the PDF, CDF, and Hazard Rate Function. The comparison revealed that fuzziness values could be estimated more precisely than real-time data.

Key Words: Fuzzy Weibull Distribution, Fuzzy Parameters, α – Cut, Probability Density Function.

1. Introduction:

The lifespan are supposed to be random. The probability distributions for random variables (lifetime density function) have precise parameters. Uncertainty and imprecise data may make determining parameters challenging. It is reasonable to presume the parameters are fuzzy quantities. Zadeh coined the term "fuzzy variable" in 1965 [1] to describe incorrect linguistic idioms and vernacular. This marked the beginning of fuzzy set theory. A fuzzy set consists of components with varying membership levels.

This work [2] provides a new method for analyzing the fuzzy system reliability of parallel-series and series-parallel systems using fuzzy confidence intervals, when the dependability of each system's component remains unknown. To compute system reliability, we estimate the dependability of each system component using fuzzy statistical data, which we model using proper tools and statistical technique. Numerical examples are provided for computing fuzzy dependability and its cut set, with the calculations performed using the R programming language.

In reliability theory, reliability measures play a critical and depreciative role in any system analysis. Measurement of reliability measurements is difficult due to the ambiguity and vagueness inherent in reliability parameters [3]. It is also challenging to incorporate a high level of uncertainty into well-established approaches and techniques. However, fuzzy logic is a useful technique for extracting exact conclusions from ambiguous and imprecise data and human perceptions. This work proposes a rule-based fuzzy logic technique to assessing dependability measurements.

The fuzzy reliability obtained by the method [4] in which the membership function of the permitted wear value is determined by the practical permitted wear value, proposed in this paper based on a practical engineering experiment, differs only slightly from conventional reliability without considering fuzziness. The top, median, and low accuracy values are 13.6%, 14.7%, and -2.3%, respectively. So we know that fuzzy reliability can be more accurate than traditional dependability at times, but not always.

The fuzzy differential equation for reliability employs two forms of fuzzy derivatives: Hukuhara derivative and generalized differentiability. It is demonstrated that the Hukuhara differentiability is insufficient for fuzzy reliability analysis. Finally, fuzzy integration will be used to illustrate the concept of fuzzy mean time to failure (FMTTF). Some numerical simulations are shown to demonstrate the applicability and validity of generalized differentiability when compared to the Hukuhara differentiability results for fuzzy reliability analysis [5].

Given its uncertainty, the failure rate is represented as a triangular fuzzy number. The fuzzy reliability is then estimated in two distinct ways using the fuzzy failure rate and the exponential life-time function for the target component, and the results are compared [6]. The first technique employs the extension concept and calculates the fuzzy dependability number appropriately. In this example, the fuzzy number of reliability is roughly triangular in shape. The second technique, which employs linear regression, involves fitting two linear functions to the right and left edges of the reliability fuzzy number, yielding a triangle fuzzy number.

In light of [7] this, in this research, we investigate the estimation of system reliability of a repairable system consisting of three identical and independent components with repair and failure rates expressed as fuzzy numbers. The uncertainty is modeled using a triangular fuzzy number. The parameters of these fuzzy numbers are estimated via confidence intervals and point estimates.

This work [8] proposes a fuzzy reliability technique based on the fuzzy universal generating function (FUGF) that considers the possible values of system variables and their corresponding probabilities as triangular fuzzy numbers (TFNs). The proposed strategy can address fuzzy dependability difficulties. A case study is used to demonstrate the proposed strategy. The design scheme's feasibility can be determined by comparing the needed reliability to the computed value.

The ideas of survival function, hazard rate function, and mean residual function were expanded for fuzzy random variables using a parametric method [9]. Following that, we develop and examine indexes for the fuzzy exponential and fuzzy two-parameter Weibull distributions, which are two often used lifetime distributions in reliability theory. Furthermore, assuming an unknown distribution of fuzzy observations (non-parametric approach), an estimate for the fuzzy distribution function is offered first, followed by an estimate for the system's ageing indices. To accomplish this, the concept of fuzzy empirical distribution function is examined as an estimate of the cumulative distribution function. Furthermore, some features of the examined estimator have been identified.

To analyze this model, define the differential equations that characterize the system states. By solving these equations, we may calculate critical characteristics like system reliability and mean time to failure (MTTF) in real-time scenarios. These metrics provide useful information on the performance and behavior of the system being studied. Using the Weibull distribution and the proposed model, this work [10] contributes to a better understanding of reliability analysis in lifetime systems and allows for the determination of critical reliability characteristics for practical applications.

This study utilized real-time data to analyze the probability density function, cumulative density function, and Hazard rate function of the fuzzy Weibull distribution with α -cuts. This study focused on Karnataka agriculture in the year 2018 [11].

2. Fuzzy Mathematical Tools:

2.1 Fuzzy Weibull Distribution:

Assume that the function $u: \mathfrak{R} \times \mathfrak{R}^n \rightarrow \mathfrak{R}$ is a partially derivable function, which means it is continuous on \mathfrak{R} and derivable except for a finite number of points that are not derived but permit later derivatives. Additionally, we have the function.

$$\begin{aligned} u &= u(x; p) \\ u &= u(x, p), x \in \mathfrak{R}, p \in \mathfrak{R}^n \\ p &= (p_1, p_2, \dots, p_n) \end{aligned}$$

Finally, u is determined by at least two parameters. Given these assumptions, Weibull's function is defined as,

$$f(x; p) = e^{-u(x;p)} \frac{du}{dx}(x; p) \quad \dots(1)$$

Furthermore, $f(x, p)$ becomes the probability density if

$$\int_{-\infty}^{\infty} f(x; p) dx = 1 \quad \dots(2)$$

The derivative is calculated at places where $u(x, p)$ is derivable, but (2) makes sense because the improper integral cannot be determined on a finite number of points. The applications encounter functions $u(x, p)$ that depend on two or three parameters, resulting in $n = 2$ or $n = 3$. In most applications, the Weibull generator contains two or three parameters, namely $p \in \mathfrak{R}_+^2$ or $p \in \mathfrak{R}_+^3$. This work will employ the situation $p = (\beta, \lambda) \in \mathfrak{R}_+^2$, and the generating function $u(x, \beta, \lambda)$ Weibull, defined as:

$$u(x, \beta, \lambda) = \begin{cases} 0, & x \leq 0 \\ \lambda x^\beta, & x > 0 \end{cases} \quad \dots(3)$$

Where $\beta, \lambda > 0$ are parameters to be determined. Accordingly Weibull probability density, biparametric, has the form:

$$f(x, \beta, \lambda) = \begin{cases} \lambda \beta x^{\beta-1} e^{-\lambda x^\beta}, & x > 0 \\ 0, & x \leq 0 \end{cases} \quad \dots(4)$$

In the first analysis, the Weibull biparametric probability density is created using the generator $u(x, \beta, \lambda)$ which is continuous on \mathfrak{R} , derivable practically everywhere on \mathfrak{R} , $\forall \lambda > 0$ and $\forall \beta > 0$. If $\beta > 1$, the Weibull generator is derivable on \mathfrak{R} . For $\beta \in (0, 1)$, we have:

$$\lim_{x \downarrow 0} u'(x) = \lim_{x \downarrow 0} \frac{\beta \lambda}{x^{1-\beta}} = \infty \quad \dots(5)$$

$$\lim_{x \uparrow 0} u'(x) = 0 \quad \dots(6)$$

We can see that $u(x, \beta, \lambda)$ meets the requirements (i) and (ii) from the Weibull generator formulation. For $f(x, \beta, \lambda)$ in (4) to be the probability density, condition (2) must also be met. This demonstrates without difficulty that:

$$\int_{-\infty}^{\infty} f(x, \beta, \lambda) dx = \left\{ \lim_{z \rightarrow \infty} (1 - e^{-z}) = 1 \right\} \quad \dots(7)$$

So

$$\int_{-\infty}^{\infty} f(x, \beta, \lambda) dx = 1 \quad \dots (8)$$

Calculate the improper integral

$$\int_{-\infty}^{\infty} f(x, \beta, \lambda) dx = \beta \lambda \int_{-\infty}^{\infty} x^{\beta-1} e^{-\lambda x^{\beta}} dx \quad \dots (9)$$

We shall leverage the change of variables.

$$z = \lambda x^{\beta}$$

$$\text{So } dz = \lambda \beta x^{\beta-1} dx \quad \dots (10)$$

This statement is significant since it was used to calculate the distribution function F(x).

$$F(x) = \int_{-\infty}^x f(t, \beta, \lambda) dt \quad \dots (11)$$

Considering (3) and (4), (11) becomes,

$$F(x) = \begin{cases} 0, & x \leq 0 \\ f(t, \beta, \lambda), & x > 0 \end{cases} \quad \dots (12)$$

$$\int_0^x \lambda \beta t^{\beta-1} e^{-\lambda t^{\beta}} dt = 1 - e^{-\lambda x^{\beta}} \quad \dots (13)$$

Thus, the distribution function F(x) is

$$F(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-\lambda x^{\beta}}, & x > 0 \end{cases} \quad \dots (14)$$

We define the function R(x, β, λ) as:

$$R(x, \beta, \lambda) = 1 - F(x) \quad \dots (15)$$

$$R(x, \beta, \lambda) = e^{-\lambda x^{\beta}} \quad \dots (16)$$

In probability theory, the distribution function is defined as follows:

$$F(x) = P(X < x) \quad \dots (17)$$

Means that the probability distribution function, namely the probability of occurrence of a continuous random variable X with density (distribution) on the Weibull model described by (4). Therefore, F(x, β, λ) negates R(x, β, λ). R(x, β, λ) is referred to as the dependability function [1]. The parameters β and λ, both positive, will be calculated using the least squares approach. For this aim, we define the function.

$$L(x, \beta, \lambda) = \ln[\ln R^{-1}(x, \beta, \lambda)] = \beta \ln x + \ln \lambda \quad \dots (18)$$

We linked a function defined by (18) to a uniform mesh with variable x>0. If we introduce the notation.

$$a = \ln \lambda, t = \ln x, y = L(x, \beta, \lambda) \quad \dots (19)$$

The expression (18) becomes:

In this fashion, the least squares regression line transforms into determination [2], which boils down to solving the system.

$$\begin{aligned} \sum_{i=1}^n (\tilde{y}_i - \beta t_i - a) t_i &= 0 \\ \sum_{i=1}^n (\tilde{y}_i - \beta t_i - a) &= 0 \end{aligned} \quad \dots (20)$$

The unknowns are β and a, with y_i and t_i representing the values associated with the uniform discretisation:

$$\begin{aligned} t_i &= \ln x_i \\ \tilde{y}_i &= \ln \left(\ln \frac{1}{p_i} \right) \end{aligned} \quad \dots (21)$$

$$\beta = \frac{C(T_n, Y_n)}{D^2(T_n)}$$

$$a = M(Y_n) - \frac{M(T_n)C(T_n, Y_n)}{D^2(T_n)} \quad \dots (22)$$

$$T_n = (t_1, t_2, \dots, t_n)^T$$

$$Y_n = (y_1, y_2, \dots, y_n)^T \quad \dots (23)$$

$$M(Y_n) = \frac{1}{n} \sum_{k=1}^n y_k \quad \dots (24)$$

$$M(Y_n T_n) = \frac{1}{n} \sum_{k=1}^n t_k y_k \quad \dots (25)$$

$$C(T_n Y_n) = M(T_n Y_n) - M(T_n)M(Y_n) \quad \dots (26)$$

$$D^2(T_n) = M(T_n^2) - [M(T_n)]^2 \quad \dots (27)$$

So

$$M(T_n^2) = \frac{1}{n} \sum_{k=1}^n t_k^2$$

In probability theory, the terms (25)-(27) have unique meaning. The intervening variables have the following meanings. The mesh with variable x > 0

$$x_1 < x_2 < \dots \dots \dots < x_n \dots \dots (28)$$

corresponds to a mesh of out-of-stock frequencies.

$$p_1 < p_2 < \dots \dots \dots < p_n \dots \dots (29)$$

2.2 Fuzzy Numbers of Weibull Type:

In the preceding paragraph, we observed that the Weibull distribution function has the form

$$F(t) = 1 - e^{-\lambda t^\beta}$$

The variable $t \in [0, T]$ indicates time. We may notice that $F(t) > 0, \forall t \geq 0$, yet $F''(t)$ has the following form:

$$F''(t) = \lambda \beta t^{\beta-2} (\beta - 1 - \lambda \beta t^\beta) e^{-\lambda t^\beta}$$

Where t^* is the inflection point of F , given by

$$t^* = \left(\frac{\beta - 1}{\lambda \beta} \right)^{\frac{1}{\beta}}$$

then the value of $F(t^*)$ is:

$$F(t^*) = 1 - e^{-\left(1 - \frac{1}{\beta}\right)}$$

Thus, we may define the Weibull member function $\mu(t)$ as:

$$\mu(t) = \begin{cases} 0, & t < 0 \\ 1 - e^{-\lambda t^\beta}, & t \geq 0 \end{cases}$$

Fuzzy cut numbers are now built for $t \geq t^*$. The construction of these numerals is as follows:

Let's take $h_0 = \mu(t^*)$, meaning h_0 is given by:

$$h_0 = 1 - e^{-\left(1 - \frac{1}{\beta}\right)}$$

The uniform network $([(\mu)]_k)_{k \geq 0}$ is given by.

$$\mu_k = h_0 + kh < 1, h > 0$$

The step $h > 0$ is decided based on the network time of the stock's issuing date. The Fuzzy number of Weibull type $I_{k,h}$ can be calculated by finding the number of cuts μ_k .

$$I_{k,h} = [t_k, t_{k+1}]$$

Where t_k is given by:

$$t_k = \left(\frac{\frac{1}{\lambda} \ln \frac{1}{1 - h_0 - kh}}{\beta} \right)^{\frac{1}{\beta}}$$

The step $h > 0$ is chosen based on the network time of the stock's issuing date. The Fuzzy number of Weibull type $I_{k,h}$ can be calculated by finding the number of cuts μ_k .

$$I_{k,h}(\alpha) = (1 - \alpha)t_k + \alpha t_{k+1}, \alpha \in [0,1]$$

3. Real Data Analysis:

The agriculture issue is the most crucial in real life. This section will assess the performance of the developed estimators on a real data set. Karnataka state agriculture in 2018. Kaggle.com [11-15] provided this data set with fuzzy membership values, which are shown in the (Table 1).

Table 1: Agriculture data of Karnataka in the year of 2018

Crop	Season	Production (kg)	Area (ha)	Yield (kg/ha)	Value (INR)
Rice	Summer	1,50,993	4,84,330	1,419.10	2,44,91,065
Safflower	Rabi	17,020	9,973	1,419.10	27,60,644
Sannhamp	Whole Year	358	85	1,419.10	58,067.60
Sesamum	Kharif	24,776	15,640	1,419.10	40,18,667
Small millets	Kharif	19,229	16,105	1,419.10	31,18,944
Soyabean	Kharif	2,61,855	2,57,221	1,419.10	4,24,72,881
Sugarcane	Whole Year	4,95,932	4,23,23,211	1,419.10	8,04,40,170
Sunflower	Kharif	95,442	81,315	1,419.10	1,54,80,692
Sunflower	Rabi	46,439	24,150	1,419.10	75,32,406
Sunflower	Summer	2,734	2,422	1,419.10	4,43,454.80
Sweet potato	Whole Year	5,032	73,050	1,419.10	8,16,190.40
Tapioca	Whole Year	635	8,439	1,419.10	1,02,997
Tobacco	Whole Year	63,840	52,442	1,419.10	1,03,54,848
Turmeric	Whole Year	26,579	1,53,767	1,419.10	43,11,114
Urad	Kharif	85,094	41,029	1,419.10	1,38,02,247
Urad	Rabi	2,787	1,484	1,419.10	4,52,051.40
Urad	Summer	1,014	521	1,419.10	1,64,470.80
Wheat	Rabi	1,57,525	1,63,113	1,419.10	2,55,50,555

Cont...

Value (INR/ha)	Total Production (kg)	Total Area (ha)	Total Yield (kg/ha)	Fuzzy Membership (Yield)
52,847.55	1,50,993	4,84,330	1,419.10	1
5,957	17,020	9,973	1,419.10	0.8
125.3	358	85	1,419.10	0.3
8,671.60	24,776	15,640	1,419.10	0.9
6,730.15	19,229	16,105	1,419.10	0.85
91,649.25	2,61,855	2,57,221	1,419.10	1
1,73,576.20	4,95,932	4,23,23,211	1,419.10	1
33,404.70	95,442	81,315	1,419.10	0.9
16,253.65	46,439	24,150	1,419.10	0.7
956.9	2,734	2,422	1,419.10	0.5
1,761.20	5,032	73,050	1,419.10	0.4
222.25	635	8,439	1,419.10	0.35
22,344.00	63,840	52,442	1,419.10	0.6
9,302.65	26,579	1,53,767	1,419.10	0.7
29,782.90	85,094	41,029	1,419.10	0.65
975.45	2,787	1,484	1,419.10	0.5
354.9	1,014	521	1,419.10	0.4
55,133.75	1,57,525	1,63,113	1,419.10	0.9

3.1 Mathematical Results of the Fuzzy Weibull Distribution with fuzzy Membership Values:

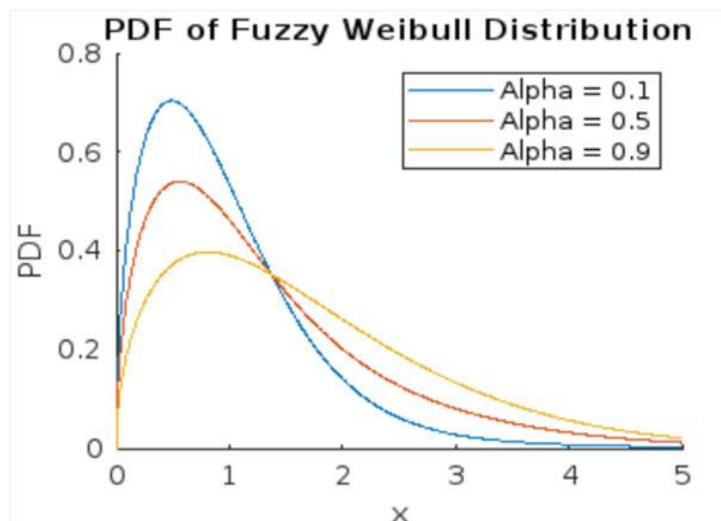


Figure 3.1: PDF of Fuzzy Weibull distribution

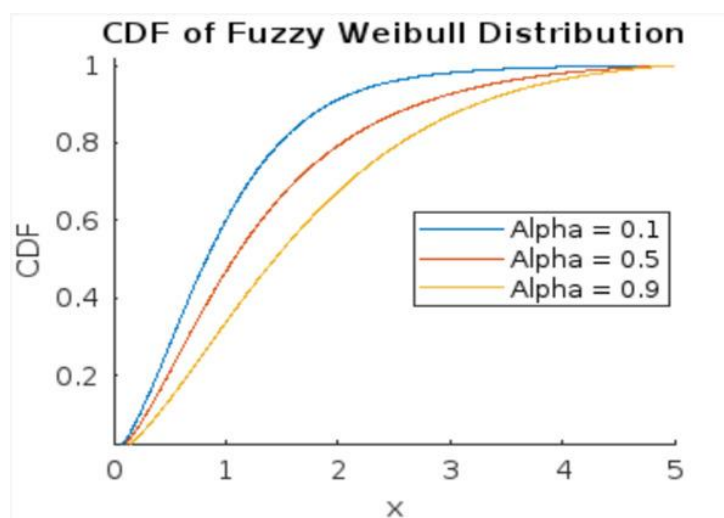


Figure 3.2: CDF of Fuzzy Weibull distribution

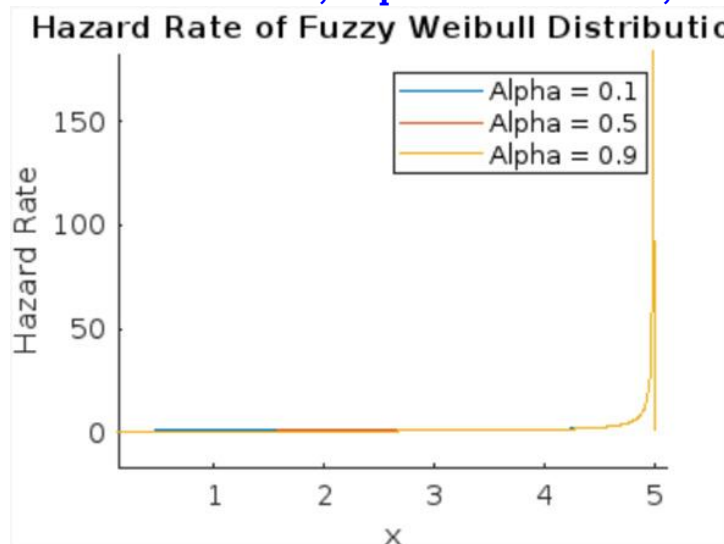


Figure 3.3: Hazard rate of Fuzzy Weibull distribution

4. Results and Discussion:

This research work makes a significant contribution to the subject of probability distributions by providing a of the Fuzzy Weibull distribution with fuzzy membership Values. The work allows us to describe a Probability density function. Cumulative density function and hazard rate function with a bathtub-shaped pattern. The bathtub-shaped hazard rate function is of tremendous importance in many disciplines because it correctly depicts the characteristics of failure rates found in real-world events. To assess the effectiveness of the newly proposed model, we conducted an empirical study with two distinct real-life temporal data sets. These agriculture data sets were carefully selected to cover a wide range of applications and ensure that the findings are generalized. Shape parameters fuzzy range defined from 0.8 to 1 and Scale parameter fuzzy range defined from 1000 to 5000. The real time agriculture data set from Kaggle.com [11-15]. Hence, we investigated crop prediction of Karnataka state use fuzzy Weibull distribution during the year 2018.

We established results for the probability density function (Figure 1), cumulative density function (Figure 2), and hazard rate function (Fig. 3) of a fuzzy Weibull distribution with fuzzy parameters. Here we demonstrate the results of all the functions given as plots using the MATLAB tool, demonstrating how well the proposed technique performs in predicting crop production for all seasons.

5. Conclusion:

This work successfully investigates fuzzy reliability and hazard functions. Reliability theory based on traditional statistical analysis may not be effective for components and parameters with random and fuzzy lifetimes. Our method for assessing fuzzy system reliability utilizes fuzzy set and probability theory. This paper treated the scaling parameter as a fuzzy membership values. Further research is needed to investigate key themes in fuzzy reliability theory, including mean residual life. This article discusses a combination of statistical methods and a fuzzy approach to Karnataka agriculture in 2018. The strategies presented in this work can assist researchers and statisticians in dealing with imprecise agricultural data during studies. Using fuzzy models to deal with fuzzy data difficulties can improve our understanding of both statistical and fuzzy approaches.

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